Multiple Choice

1 (B) $\frac{10}{2}\times\frac{-6}{2}=-15$

2 (A) $\tan\theta=\frac{5-3}{1+5\times3}=\frac{2}{16}=\frac{1}{8}$

3 (A) $\lim_{x\to0}\frac{\sin3x}{3x}\times\frac{\cos3x}{4}=\frac{1}{4}$

4 (D) $b=2, c=1, d=-1,$ and when $x=0, a\times2\times1\times1=-6,$

$\therefore a=-3$

5 (A) When $t=0, P=3000,$ when $t\to\infty, P\to1500$

6 (C) The tangent at $x=c$ hits the x-axis closer to w

7 (C) $a=\frac{d}{dx}\left(\frac{1}{2}v^2\right)=\frac{d}{dx}\left(\frac{1}{2}(x^2+2)^2\right)=2x(x^2+2)=6$

when $x=1$

8 (B) 5!6!

9 (D) $2x=(-1)^n-\frac{\pi}{6}+n\pi,\therefore x=(-1)^{n+1}\frac{\pi}{12}+\frac{n\pi}{2}$

10 (B) $v^2=-n^2\left(k^2-(x-k)^2\right),\therefore$ amplitude $=k,$ centre $x=k$

Question 11

(a) (i) $P(1)=1-2-5+6=0,\therefore x=1$ is a zero

(ii) $x^3-2x^2-5x+6=(x-1)(x^2-x-6)$

$=(x-1)(x-3)(x+2),\therefore$ Other zeros are -2 and 3

(b) $5(x-2)=2^3,\therefore x=2+\frac{8}{5}=\frac{18}{5}$

(c) $\sqrt{3}\sin x+\cos x=2\sin\left(x+\frac{\pi}{6}\right)$

(d) $x(x+2)=3\times8,\therefore x^2+2x-24=(x+6)(x-4)=0,$

$\therefore x=4$

(e) (i) $x\neq\frac{1}{4}$

(ii) $\frac{1}{4x-1}<1$

$4x-1<(4x-1)^2$

$(4x-1)(4x-2)>0$

$x<\frac{1}{4}$ or $x>\frac{1}{2}$

(f) Let $u^2=1-x, 2udu=-dx$

When $x=-3, u=2.$ When $x=0, u=1$

$\int_{-3}^{0}\frac{x}{\sqrt{1-x}}dx=\int_{1}^{2}\frac{1-u^2}{u}2udu=2\left[u-\frac{u^3}{3}\right]_1^2$

$=2\left(-\frac{2}{3}-\frac{2}{3}\right)=-\frac{8}{3}$

Question 12

(a) $\int \cos^2 3x dx = \int \frac{1+\cos 6x}{2} dx = \frac{x}{2} + \frac{\sin 6x}{12} + C$

(b) (i) $\sin\theta = \frac{h}{20}, \therefore h = 20\sin\theta, \therefore \frac{dh}{d\theta} = 20\cos\theta$

(ii) $\frac{dh}{dt} = \frac{dh}{d\theta}\frac{d\theta}{dt} = 20\cos\theta \times 1.5 = 30\cos\theta$

When $h = 15, \sin\theta = \frac{15}{20} = \frac{3}{4}, \therefore \cos\theta = \frac{\sqrt{7}}{4}$

$\therefore \frac{dh}{dt} = \frac{15\sqrt{7}}{2}$ m/m

(c) (i) $f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$

(ii) Since $f'(x) = 0$, $f(x)$ is a constant

Let $x = 0$, $f(0) = \frac{\pi}{2}, \therefore f(x) = \frac{\pi}{2}$

(iii) It's simply a horizontal line at $y = \frac{\pi}{2}$ for $-1 \le x \le 1$

(d) Pr(at least 10) = ${}^{12}C_{10} 0.75^{10} 0.25^2 + {}^{12}C_1 0.75^{11} 0.25 + 0.75^{12}$

(e) (i) The equation of PT is $y = px - ap^2, \therefore A(ap, 0)$

$m_{SA} = \frac{a}{-ap} = -\frac{1}{p}$ while $m_{PT} = p$

$m_{SA} \times m_{PT} = -1, \therefore \angle PAS = 90°$

(ii) Similarly, $\angle QBS = 90°, \therefore SBAT$ is a cyclic quad because angles subtending the same arc are equal

(iii) The diameter is $TS = \sqrt{\left(a(p+q)^2 + (apq-a)^2\right)}$

$= a\sqrt{(p+q)^2 + (pq-1)^2}$

$= a\sqrt{p^2 + q^2 + 2pq + p^2q^2 - 2pq + 1}$

$= a\sqrt{p^2 + q^2 + p^2q^2 + 1}$

$= a\sqrt{(p^2+1)(q^2+1)}$

Question 13

(a) Let $n = 1$, LHS $= 2$, RHS $= \frac{1+3}{2} = 2, \therefore$ true

Assume $2 - 6 + ... + 2(-3)^{n-1} = \frac{1-(-3)^n}{2}$ for some value of n

RTP $2 - 6 + ... + 2(-3)^{n-1} + 2(-3)^n = \frac{1-(-3)^{n+1}}{2}$

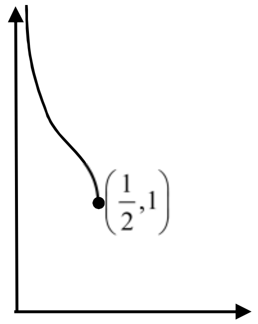
LHS $= \frac{1-(-3)^n}{2} + 2(-3)^n = \frac{1-(-3)^n + 4(-3)^n}{2} = \frac{1+(-3)^n}{2}$

$= \frac{1-(-3)^{n+1}}{2} =$ RHS.

$\therefore$ By the PMI, it is true for all $n \ge 1$

(b) (i) D: $0 < x \le \frac{1}{2}$, R: $y \ge 1$

(ii) See graph



(iii) $x = \frac{y}{y^2+1}, \therefore xy^2 - y + x = 0, \therefore y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$

Take $f^{-1}(x): y = \frac{1+\sqrt{1-4x^2}}{2x}$ as $y \ge 1$

(c) (i) Let $y = 0, t = \frac{2V\sin\theta}{g}$,

$\therefore x = \frac{2V^2\sin\theta\cos\theta}{g} = \frac{V^2}{g}\sin 2\theta$

(ii) $x = \frac{V^2}{g}\sin 2\left(\frac{\pi}{2} - \theta\right) = \frac{V^2}{g}\sin(\pi - 2\theta) = \frac{V^2}{g}\sin 2\theta$

(iii) When $t = \frac{V\sin\theta}{g}$,

$y = \frac{V^2\sin^2\theta}{g} - \frac{V^2\sin^2\theta}{2g} = \frac{V^2\sin^2\theta}{2g}$

$\therefore h_\alpha = \frac{V^2\sin^2\alpha}{2g}, h_\beta = \frac{V^2\sin^2\left(\frac{\pi}{2}-\alpha\right)}{2g} = \frac{V^2\cos^2\alpha}{2g}$

$\therefore \frac{h_\alpha + h_\beta}{2} = \frac{V^2}{4g}\left(\sin^2\alpha + \cos^2\alpha\right) = \frac{V^2}{4g}$, thus it depends only on V and g

Question 14

(a) $\angle SPQ = \angle APB$ (vertically opposite)

but $\angle APB = 180° - (\alpha + \beta)$ (angle sum in a Δ)

$\therefore \angle SPQ = 180° - (\alpha + \beta)$.

Similarly, $\angle SRQ = 180° - (\gamma + \delta)$

$\therefore \angle SPQ + \angle SRQ = 360° - (\alpha + \beta + \gamma + \delta)$

but $2(\alpha + \beta + \gamma + \delta) = 360°$ (angle sum of a quad)

$\therefore \angle SPQ + \angle SRQ = 180°$

$\therefore PQRS$ is a cyclic quad (opposite angles are supplementary)

(b) (i) $\binom{n}{r}2^{n-r}$ is the coefficient of x^r in the RHS.

$$\left(1+(1+x)^n\right) = 1 + \binom{n}{1}(1+x) + \binom{n}{2}(1+x)^2 + \ldots$$
$$+\binom{n}{r}(1+x)^r + \binom{n}{r+1}(1+x)^{r+1} + \ldots + \binom{n}{n}(1+x)^n$$

$\therefore$ The coefficients of x^r in the LHS is

$$\binom{n}{r}\binom{r}{r} + \binom{n}{r+1}\binom{r+1}{r} + \ldots + \binom{n}{n}\binom{n}{r} \qquad (*)$$

(ii) Let $n = 23, r = 4,$ the line marked (*) becomes

$$\binom{23}{4}\binom{4}{4} + \binom{23}{5}\binom{5}{4} + \ldots + \binom{23}{23}\binom{23}{4}$$

i.e. selecting 4 for selector A (then 4 for B), or 5 for A (then 4 for B), or 6 for A (then 4 for B) and so on.

From part (i), the total $= \binom{23}{4}2^{23-4} = \binom{23}{4}2^{19}$

(c) (i) In ΔABC and ΔACD

$\angle A$ is common

$\angle ACB = \angle ADC = 90°$

$\therefore \Delta ABC$ and ΔACD are similar (AA)

(ii) Area of $\Delta ABC = \frac{1}{2}ab = \frac{1}{2}xc$

$\therefore x = \frac{ab}{c}$

(iii) $\Delta AFH \parallel \Delta ACD$ ($\angle A$ is common and $\angle F = \angle D$ $= 90°$), $\therefore \frac{x_1}{x} = \frac{AF}{AC} = \frac{b-x}{b} = \frac{b - \frac{ab}{c}}{b} = \frac{c-a}{c}$.

$\therefore x_1 = \left(\frac{c-a}{c}\right)x$

Similarly, $x_2 = \left(\frac{c-a}{c}\right)^2 x, x_3 = \left(\frac{c-a}{c}\right)^3 x,$ and so on.

Total area $= \frac{\pi}{4}\left(x^2 + x_1^2 + x_2^2 + \ldots\right)$

$$= \frac{\pi}{4}\left(x^2 + \left(\frac{c-a}{c}\right)^2 x^2 + \left(\frac{c-a}{c}\right)^4 x^2 + \ldots\right)$$

$$= \frac{\pi}{4}\left(\frac{a^2b^2}{c^2}\right)\frac{1}{1-\left(\frac{c-a}{c}\right)^2}$$

$$= \frac{\pi}{4}\left(\frac{a^2b^2}{c^2}\right)\frac{c^2}{c^2-(c-a)^2}$$

$$= \frac{\pi}{4}\frac{a^2b^2}{2ac-a^2}$$

$$= \frac{\pi ab^2}{4(2c-a)}$$

(iv) Area of all the quadrants < area of ΔABC

$$\frac{\pi ab^2}{4(2c-a)} < \frac{1}{2}ab$$

$$\therefore \frac{\pi}{2} < \frac{2c-a}{b}.$$